“Musing on exponentiation in groups”

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To Vladimir Remeslennikov, my dear teacher, a partner in ourmath endeavors, and a friend, on occasion of his 85th birthday.

Exponentiation in groups is an old and well-researched subject. The main theme here is to understand what a “non-commutative module” is in various classes of groups. Following Lyndon in 1994 V. Remeslennikov and myself introduced a notion of a group admitting exponentiation in an associative unitary ring R (now called R-groups). This is the most “freest and universal” exponentiation that works in all groups and it applies nicely to free and hyperbolic groups, free products with amalgamation and HNN extensions, etc. M. Amaglobeli started studying R-groups in a variety. However, if a group satisfies an identity the notion of exponentiation can be further adjusted to reflect more closely the nature of the group. Thus, in the class of nilpotent groups there is famous P. Hall and A. Mal’cev’s exponentiation that gives a perfect notion of a “nilpotent non-commutative module”. Recently, working on first-order properties of free metabelian groups, we together with O. Kharlampovichexplored an exponentiation that naturally occurs in metabelian groups. In this talk I will discuss all these exponentiations, the corresponding centroids and tensor completions, and how they relate to each other.