

Quantisation ideals: multi-quantum systems and non-deformation quantisation

Abstract

We propose to reformulate the problem of quantisation and look at it from an entirely new angle, focussing on quantisation of dynamical systems themselves, rather than of their Poisson structures. We begin with a lift of a classical dynamical system to a system on a free associative algebra with non-commutative dynamical variables reducing the problem of quantisation to the problem of searching two-sided ideals (quantisation ideals) in the free algebra that define the commutation relations of dynamical variables and are invariant with respect to the non-commutative dynamics. Quantum multiplication rules in the quotient algebra over a quantisation ideal are manifestly associative and consistent with the dynamics. We found first examples of bi-quantum systems which are quantum counterparts of bi-Hamiltonian systems in the classical theory. Moreover, the new approach enables us to define and present first examples of non-deformation quantisations of dynamical systems. In the “classical” limit it yields Poisson algebras associated with deformations of commutative or non-commutative algebras. The new approach sheds light on the problem of operator’s ordering.

References

- [1] A.V. Mikhailov. Quantisation ideals of nonabelian integrable systems. *Russ. Math. Surv.*, 75(5):199, 2020.
- [2] V.M. Buchstaber and A.V. Mikhailov. KdV hierarchies and quantum Novikov’s equations. arXiv:2109.06357.
- [3] S. Carpentier, A.V. Mikhailov and J.P. Wang. Quantisation of the Volterra hierarchy. *Lett. Math. Phys.*, 112:94, 2022.